cut-tree diagram illustrates every possible sequence of cuts for two players carrying out CNH for the ratio of 9:4:

![Cut-Tree Diagram illustrating the Cut-Near-Halves algorithm for a 9:4 ratio. Alice is to receive 9/13 of the cake, and Bob 4/13. Bob first cuts the cake into two pieces, worth 7/13 and 6/13. (We omit the denominators for simplicity.) Alice chooses one of these, and so she requires either 2/13 or 3/13 more, depending on her choice. This process continues until one of the players has received their due portion, at which point the other player takes the remaining cake.](image)

**B. Remarks**

The figure shows that for a split ratio of 9:4, CNH may take up to 4 cuts. Moreover, it can be shown that the number of cuts necessary for a division of \(a:b\) approaches \(\lceil \log_2(a+b) \rceil\) [5]. This is because the cake is being cut approximately in half at each step. Indeed, less cuts than \(\lceil \log_2(a+b) \rceil\) will often suffice, however, if interested in the worst case scenario for which \(\lceil \log_2(a+b) \rceil\) cuts is a good working estimate. It is noted that the mechanism behind the success of CNH as opposed to other methods: when the cutter proposes two (near-) halves to the chooser, the chooser is guaranteed to agree that one of them is worth at least (near-) half of the cake, thereby reducing the subsequent ratio sum by (nearly) a factor of 2. Suppose the cutter offers portions other than halves to the chooser as is the case when dividing by Ramsey Partitions [1]. Then if the chooser disagrees with the larger portion and instead selects the smaller portion, the subsequent ratio sum has not been reduced by a factor of 2. CNH seems to reduce the resultant ratio sum by the greatest amount, independent of the chooser’s selection. The only way to improve upon the algorithm would be to somehow guarantee that after each cut; the resultant ratio sum reduces by greater than a factor of 2.

**IV. A New Algorithm**

**A. Initial Strategy and an Example**

It has been recognized that in order to improve upon CNH, some factorization of resultant ratio sums must be achieved. An additional fact will accompany this idea. If the player who is owed less of the cake cuts a certain portion which is less than the amount the player is owed, then either player may take that portion and the integrity of the problem is still upheld. To demonstrate this through an example, consider again the ratio of 9:4. Suppose Bob cuts 1/13 and proposes that Alice take it. Whether or not Alice takes it, Bob believes the rest of the cake to be worth 12/13, so he is satisfied. Perhaps Alice disagrees, and Bob takes the 1/13. By virtue of rejecting the 1/13, Alice believes the remaining cake is at least 12/13. Since Bob cut the 1/13, he believes the rest is exactly 12/13. Alternatively, if Alice takes the piece, she must believe it is worth \(x/13\), with \(x > 1\). The problem remains linear, and so after the division of the remaining cake, Alice will likely have more than her 9/13 requirement.

Can a portion that would guarantee a factorization of the subsequent ratio be found? More formally, for a given ratio of \(a:b\), seeking a portion \(x\), with \(x < b\), such that if the chooser takes the \(x\) portion, then the fraction \((a-x)/b\) reduces by some factor, and likewise if the cutter takes it, the fraction \(a/(b-x)\) reduces by some (other) factor. Here, the worst case is the scenario where the subsequent ratio sum reduces the least. Suppose such a cut can be...